

THE UNIFIED MODEL OF ELECTROWEAK INTERACTIONS AND APPLICATION TO NEUTRINO PHENOMENOLOGY

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Abstract

The unified model of electroweak interactions (Glashow-Weinberg-Salam model) is constructed step by step. As an application, the phenomenology of neutrino mixing, in the framework of the three-family model, is discussed in detail. Numerous formulae for appearance or disappearance of neutrinos in vacuum or in matter are derived. These notes should provide a self-contained introduction to the GWS model and to neutrino mixing and oscillations. Recent experimental results on neutrino oscillations are briefly reviewed.

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The following notes are rather detailed so that a student who does not have a proper academic environment in particle physics can find them self-sufficient. The prerequisite is a course on advanced quantum mechanics and some knowledge on the notion of invariant scattering amplitudes and Feynman rules for fermions and bosons.

These notes are available at the following URL : https://lectures.lapth.cnrs.fr/standard_model where some exercices on the calculation of particle production and decay can be found.

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1 Introduction

From the experimental point of view the world of “elementary particles” consists in:

- *leptons*: they have spin $\frac{1}{2}$ and come in three doublets, (e^-, ν_e) the electron and its associated neutrino, (μ^-, ν_μ) the muon and its neutrino, (τ^-, ν_τ) the tau and its neutrino.
- *vector bosons*: they have spin 1 and there is a massless boson, the photon, and three massive ones, the W^+ , W^- and the Z .
- *hadrons*: one distinguishes mesons of integer spin ($S = 0, 1, \dots$) from baryons of half-integer spin ($S = \frac{1}{2}, \frac{3}{2}, \dots$). The *hadrons* have been known for a long time to have a finite size (typically of the order of 1 fm) and there exist so many hadrons (about 150 mesons and 120 baryons) that they cannot be considered as elementary. At high energy they appear as composite objects made up of quasi-free point-like fields : the quarks and the gluons. Like the *leptons* and the *vector bosons*, the quarks and the gluons are structureless down to a scale of about 10^{-3} to 10^{-4} fermi, *i.e.* 10^{-18} to 10^{-19} m according to the most recent experimental results obtained at the CERN Large Hadron Collider (LHC). They are treated as elementary fields appearing in the lagrangian which describes the dynamics of their interactions.

Three types of forces have been identified acting on these fields: the strong force which affects only the quarks and the gluons, and the electromagnetic and weak forces. The basic principle which guides the construction of models of particle physics is that of local gauge invariance according to which the physical properties do not depend on the phases of the fields. The Standard Model is a (highly successful) example of a minimal model based on the local gauge group

$$SU(3) \otimes SU(2)_L \otimes U(1)_Y$$

i.e. the direct product of three simple groups. The main features of these groups are :

- The $SU(3)$ gauge group or colour group is the symmetry group of strong interactions. This group acts on the quarks and the interaction force is mediated by the gluons which are the gauge bosons of the group. The quarks and the gluons are coloured fields. The “coupling” (fine structure constant) between quarks and gluons is denoted by α_s which can be large. Under some conditions, however, α_s becomes very small and perturbation theory applies. The $SU(3)$ colour symmetry is exact and consequently the gluons are massless. The theory of strong interactions based on colour $SU(3)$ is called Quantum Chromodynamics ;
- The $SU(2)_L \otimes U(1)_Y$ is the gauge group of the unified weak and electromagnetic interactions, where $SU(2)_L$ is the weak isospin group, acting on left-handed fermions, and $U(1)_Y$ is the hypercharge group. At “low” energy, below 250 GeV, the $SU(2)_L \otimes U(1)_Y$ symmetry is “spontaneously” broken

and the residual group is $U(1)_{\text{em}}$ whose generator is a linear combination of the $U(1)_Y$ generator and a generator of $SU(2)_L$: the corresponding gauge boson is of course the photon and the associated “coupling” is $\alpha \simeq \frac{1}{137}$. Symmetry breaking implies that the other gauge bosons acquire a mass: they are the heavy $W^\pm, Z$ bosons discovered at CERN in the mid ’80’s. The symmetry breaking mechanism is associated to the names of Brout, Englert, Higgs, Guralnik, Hagen, Kibble and Sudarshan and it is known now as the BEH mechanism after the names of the authors (Brout, Englert, Higgs) who published their results first. Higgs emphasized the existence of a massive scalar field as a consequence of the spontaneously broken symmetry and this field is traditionally referred to as the Higgs boson, but it is sometimes called also the “BEH boson” or simply the “massive scalar boson”. Unlike the strong and electromagnetic interactions, the weak interactions violate parity. The electroweak theory, based on spontaneously broken $SU(2)_L \otimes U(1)_Y$ gauge invariance, is known as the Glashow-Salam-Weinberg (GSW) model.

A specific feature of the electroweak model is the generation mixing occurring at the Born level independently for the quark and the lepton sectors. The corresponding formalism is associated to the Cabibbo-Kobayashi-Maskawa (CKM) matrix for the quarks and the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix for the leptons. In the latter case the consequences are neutrino oscillations the phenomenology of which will be the object of the second part of these notes.

Before entering the description of the unified theory of electroweak interactions, based on broken gauge invariance, it is useful to briefly review the Fermi theory of weak interactions and its phenomenological extensions: this will serve to motivate the choice of the gauge group $SU(2)_L \otimes U(1)_Y$ as well as illustrate the features related to the presence of massive gauge bosons.