10 Exercises: Higgs boson decays

We consider in the following various two body decays of a Higgs boson.

10.1 Kinematics

Let the Higgs boson of mass M_H and momentum q decay into particles A and B of masses m_1 and m_2 and momenta p_1 and p_2 respectively: $H(q) \to A(p_1) + B(p_2)$. The decay rate summed over final polarisations and colours is:

$$d\,\Gamma = \frac{1}{2\,M}\,\frac{d^3p_1}{(2\,\pi)^3\,2\,E_1}\,\frac{d^3p_2}{(2\,\pi)^3\,2\,E_2}\,(2\,\pi)^4\,\delta^4\,(q-p_1-p_2)\,\left|\bar{T}\right|^2\ , \tag{10.1}$$

with $\left|\bar{T}\right|^2$ the invariant matrix element squared, summed over final colours and polarisations. Momentum conservation imposes $p_1.p_2 = (M_H^2 - m_1^2 - m_2^2)/2$ with $p_1^2 = m_1^2$ et $p_2^2 = m_2^2$. Thus $\left|\bar{T}\right|^2$ depends only on the external masses $\left|\bar{T}(m_1^2, m_2^2, M_H^2)\right|^2$ and the integral in eq. (10.1) can be done independently of the decay channel. Using $d^3p_2/2\,E_2 = d^4p_2\,\delta^+(p_2^2 - m_2^2)$ and carrying out the d^4p_2 integration it comes out

$$d\,\Gamma = \frac{1}{2\,M_H}\,\frac{\left|\bar{T}\right|^2}{(2\,\pi)^2}\int\frac{d^3p_1}{2\,E_1}\,\delta^+\left((q-p_1)^2 - m_2^2\right). \tag{10.2}$$

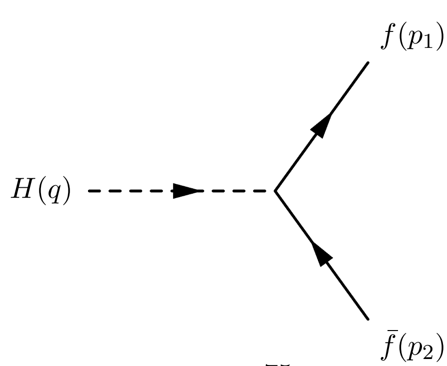
Going to the rest frame of the Higgs boson, $q = (M, 0, 0, 0)$, one finds that the argument of the δ^+ function reduces to $(M^2 - 2ME_1 + m_1^2 - m_2^2)$ independent of the angles. Since all cases we consider have $m_1 = m_2$ the expressions will simplify. Using $p_1\,dp_1 = E_1\,dE_1$ all integrations are easily done to get:

$$\boxed{\Gamma = \frac{1}{16\,\pi\,M_H}\,\left|\bar{T}\right|^2\,\sqrt{1 - \frac{4\,m^2}{M^2}}\ ,} \tag{10.3}$$

with m the common mass of the decay products.

10.2 Higgs decay into a fermion anti-fermion pair

This channel has only one diagram with the Higgs fermion-antifermion coupling, m_f/v given in eq. (8.29):



The coresponding amplitude T is:

$$T = -i\,\frac{m_f}{v}\,\bar{u}(p_1)\,v_(p_2) \tag{10.4}$$

leading to:

$$\begin{aligned} |\bar{T}|^2 &= \frac{m_f^2}{v^2}\,(N)\,\left(Tr\,[\not p_1\,\not p_2] - m_f^2\,Tr[1]\right) \\ &= \frac{2\,m_f^2\,M_H^2}{v^2}\left(1 - \frac{4\,m_f^2}{M_H^2}\right)(N)\ . \end{aligned} \tag{10.5}$$

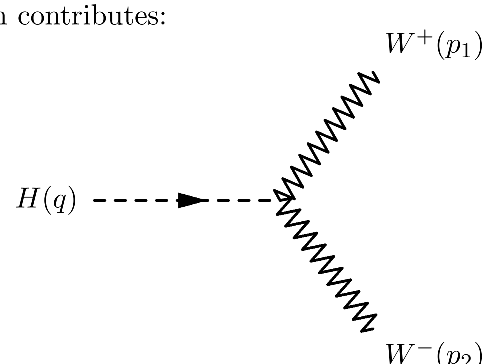
In the above result the colour factor N has been put in parentheses to indicate that, if the final fermions are quarks then we keep this factor, while if they are leptons it should be ignored. Getting rid of the vacuum expectation value v in favor of physical quantities via eq. (8.19), $1/v = e/(2\sin\theta_W\,M_W)$ and $e^2 = 4\pi\alpha$ the decay rate is:

$$\boxed{\Gamma_{H\to f\,\bar{f}} = \frac{(N)\,\alpha}{8\,\sin^2(\theta_W)}\,\frac{M_H\,m_f^2}{M_W^2}\left(1 - \frac{4\,m_f^2}{M_H^2}\right)^{3/2}} \tag{10.6}$$

where eq. (10.3) has been used.

10.3 Higgs decay into a $W^+\,W^-$ pair

Here again only one diagram contributes:



The amplitude for this transition is:

$$T = i\,\frac{e\,M_W}{\sin\theta_W}\,g^{\alpha\,\beta}\,\varepsilon_\alpha^{\lambda_1 *}(p_1)\,\varepsilon_\beta^{\lambda_2 *}(p_2)\ , \tag{10.7}$$

with $\varepsilon_\alpha^\lambda(p)$ the polarisation vector of a gauge boson and the coupling given in eq. (8.22). The sum over polarisations is done using:

$$\sum_\lambda \varepsilon_\alpha^\lambda(p)\,\varepsilon_\beta^{\lambda *}(p) = -g_{\alpha\,\beta} + \frac{p_\alpha\,p_\beta}{M_W^2}\ , \tag{10.8}$$

so that:

$$
\begin{aligned}
|\bar{T}|^2 &= \left(\frac{e\,M_W}{\sin\theta_W}\right)^2 \left(-g_{\alpha\,\mu} + \frac{p_{1\,\alpha}\;p_{1\,\mu}}{M_W^2}\right)\left(-g^{\alpha\,\mu} + \frac{p_2^{\alpha}\;p_2^{\mu}}{M_W^2}\right) \\
&= \frac{e^2}{4\,\sin^2\theta_W\,M_W^2}\left(12\,M_W^4 + M_H^4 - 4\,M_W^2\,M_H^2\right)\;. \qquad (10.9)
\end{aligned}
$$

Finally the decay rate is:

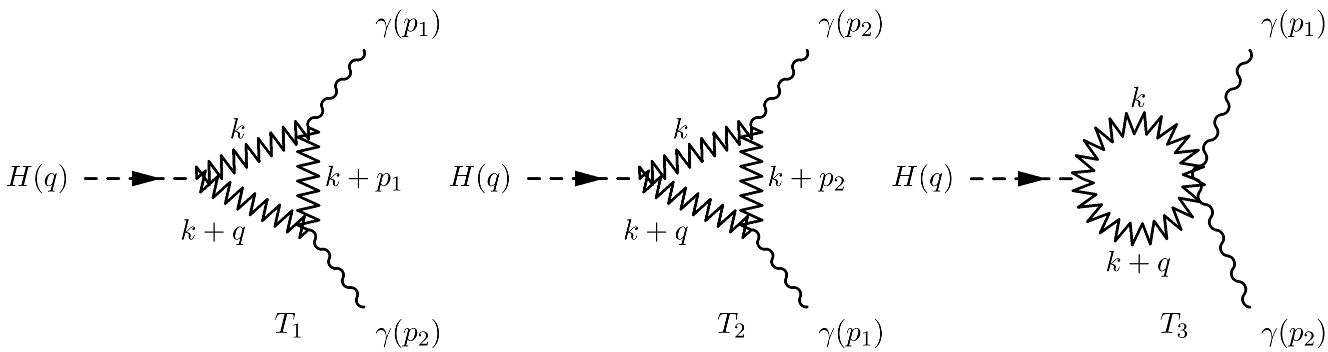
$$
\boxed{\Gamma_{H\to W^+\,W^-} = \frac{\alpha}{16\,\sin^2(\theta_W)}\,\frac{M_H^3}{M_W^2}\,\sqrt{1-\frac{4\,M_W^2}{M_H^2}}\left(1-4\,\frac{M_W^2}{M_H^2}+12\,\frac{M_W^4}{M_H^4}\right)} \qquad (10.10)
$$

10.4 Higgs decay in a $\gamma\;\gamma$ pair

As seen in sec. 8.4 this transition goes via two types of loop diagrams, one involving fermions and the other charged gauge bosons.

10.4.1 W boson loop

In the unitary gauge three types of diagrams contribute:



All these diagrams have a common structure, namely the HWW vertex and the two adjacent W propagators of momentum k and $k{+}q$ respectively. Each amplitude T_i is written as $T_i = T_i^{\mu_1\,\mu_2}\,\varepsilon^*_{\mu_1}(p_1)\,\varepsilon^*_{\mu_2}(p_2)$ where we drop for simplicity the photon polarisation indices. Furthermore we introduce the tensor $\tilde{T}_i^{\mu_1\,\mu_2}$:

$$
T_i^{\mu_1\,\mu_2} = \int \frac{d^n k}{(2\,\pi)^n}\,\tilde{T}_i^{\mu_1\,\mu_2}\;,
$$

with $\tilde{T}_i^{\mu_1\,\mu_2} = R_{\alpha_1\,\alpha_2}\,M_i^{\mu_1\,\mu_2\,\alpha_1\,\alpha_2}$, $R_{\alpha_1\,\alpha_2}$ containing the part common to all three digrams. Applying the Feynman rules, using the unitary gauge for the W propagators, it comes out:

$$
\begin{aligned}
M_1^{\mu_1\,\mu_2\,\alpha_1\,\alpha_2} &= (i\,e)\,\Big[g^{\alpha_2\,\mu_2}\,(k+q+p_2)^{\beta_1} + g^{\mu_2\,\beta_1}\,(-p_2+k+p_1)^{\alpha_2} + g^{\beta_1\,\alpha_2}\,(-k-p_1-k-q)^{\mu_2}\Big] \\
&\quad \times (-i)\,\left(g_{\beta_1\,\beta_2} - \frac{(k+p_1)_{\beta_1}\,(k+p_1)_{\beta_2}}{M_W^2}\right)\,\frac{1}{(k+p_1)^2 - M_W^2 + i\,\epsilon} \\
&\quad \times (i\,e)\,\Big[g^{\beta_2\,\mu_1}\,(k+p_1+p_1)^{\alpha_1} + g^{\mu_1\,\alpha_1}\,(-p_1+k)^{\beta_2} + g^{\alpha_1\,\beta_2}\,(-k-k-p_1)^{\mu_1}\Big] \quad (10.11) \\
M_2^{\mu_1\,\mu_2\,\alpha_1\,\alpha_2} &= M_1^{\mu_1\,\mu_2\,\alpha_1\,\alpha_2}(\mu_1 \leftrightarrow \mu_2, p_1 \leftrightarrow p_2) \quad (10.12) \\
M_3^{\mu_1\,\mu_2\,\alpha_1\,\alpha_2} &= i\,e^2\,\left[g^{\alpha_1\,\mu_1}\,g^{\alpha_2\,\mu_2} + g^{\alpha_1\,\mu_2}\,g^{\alpha_2\,\mu_1} - 2\,g^{\mu_1\,\mu_2}\,g^{\alpha_1\,\alpha_2}\right]\ , \quad (10.13)
\end{aligned}
$$

and for the common structure of the amplitudes:

$$
\begin{aligned}
R_{\alpha_1\,\alpha_2} &= -i\,\frac{e\,M_W}{\sin(\theta_W)}\,\left(g_{\alpha_1\,\alpha_2} - \frac{(k+q)_{\alpha_1}\,(k+q)_{\alpha_2}}{M_W^2} - \frac{k_{\alpha_1}\,k_{\alpha_2}}{M_W^2} + \frac{k_{\alpha_1}\,(k+q)_{\alpha_2}\,k.(k+q)}{M_W^4}\right) \\
&\quad \times \frac{1}{(D_0 + i\,\epsilon)\,(D_3 + i\,\epsilon)}\ . \quad (10.14)
\end{aligned}
$$

The quantities D_0 and D_3 are the denominators of propagators,

$$D_0 = k^2 - M_W^2\ , \qquad D_3 = (k+q)^2 - M_W^2\ , \quad (10.15)$$

and we will need later,

$$D_1 = (k+p_1)^2 - M_W^2\ , \qquad D_2 = (k+p_2)^2 - M_W^2\ . \quad (10.16)$$

The diagrams T_1, T_2 et T_3 are highly divergent in the ultraviolet region:

$$
\begin{aligned}
T_1 \,\text{et}\, T_2 &\simeq \textstyle\int d^4k\,\frac{k^8}{k^6} \simeq \int dk\,k^5 \\
T_3 &\simeq \textstyle\int d^4k\,\frac{k^6}{k^6} \simeq \int dk\,k^3\ ,
\end{aligned}
$$

but working in n space-time dimensions regularizes the divergencies. Rather than evaluating these integrals by brute force we try to arrange the terms to make possible cancellations obvious in the integrands. One thus defines:

$$
\begin{aligned}
T^{\mu_1\,\mu_2} &= T_1^{\mu_1\,\mu_2} + T_2^{\mu_1\,\mu_2} + T_3^{\mu_1\,\mu_2} \\
&= \int \frac{d^n k}{(2\,\pi)^n}\,\left(\tilde{T}_1^{\mu_1\,\mu_2} + \tilde{T}_2^{\mu_1\,\mu_2} + \tilde{T}_3^{\mu_1\,\mu_2}\right)\ . \quad (10.17)
\end{aligned}
$$

After integration on the loop momentum k the tensor $T^{\mu_1\,\mu_2}$ depends only on the external momenta p_1, p_2 and it can be parameterised as:

$$T^{\mu_1\,\mu_2} = \frac{A}{p_1.p_2}p_1^{\mu_2}\,p_2^{\mu_1} + \frac{B}{p_1.p_2}\,p_1^{\mu_1}\,p_2^{\mu_2} + C\,g^{\mu_1\,\mu_2}\ . \quad (10.18)$$

The aim is to calculate the expressions A, B and C. For this purpose we construct the following scalars:

$$
\begin{aligned}
g_{\mu_1\,\mu_2}\,T^{\mu_1\,\mu_2} &= A + B + n\,C \\
p_{1\,\mu_2}\,p_{2\,\mu_1}\,T^{\mu_1\,\mu_2} &= p_1.p_2\,(B + C) \\
p_{1\,\mu_1}\,p_{2\,\mu_2}\,T^{\mu_1\,\mu_2} &= p_1.p_2\,(A + C)\ ,
\end{aligned}
$$

where the property $g_{\mu_1\,\mu_2}\,g^{\mu_1\,\mu_2} = n$ has been used since we work in n dimensions. The system of equations is easily solved to find:

$$
C = \frac{1}{2\,(n-2)}\left(g_{\mu_1\,\mu_2}\,T^{\mu_1\,\mu_2} - \frac{p_{1\,\mu_2}\,p_{2\,\mu_1}}{p_1.p_2}\,T^{\mu_1\,\mu_2} - \frac{p_{1\,\mu_1}\,p_{2\,\mu_2}}{p_1.p_2}\,T^{\mu_1\,\mu_2}\right) \tag{10.19}
$$

$$
B = \frac{p_{1\,\mu_2}\,p_{2\,\mu_1}}{p_1.p_2}\,T^{\mu_1\,\mu_2} - C \tag{10.20}
$$

$$
A = \frac{p_{1\,\mu_1}\,p_{2\,\mu_2}}{p_1.p_2}\,T^{\mu_1\,\mu_2} - C\ . \tag{10.21}
$$

The various contractions of the tensor $T^{\mu_1\,\mu_2}$ are calculated with the help of a `form` program[16,17]. By reconstructing systematically the quantities $D_0, \cdots, D_3$ in the numerators and cancelling them with the denominators, we get rid of the k dependence in the numerators so that only scalar integrals have to be evaluated. There are two 3-point integrals:

$$
\begin{aligned}
I_{013} &= \int \frac{d^n k}{(2\,\pi)^n}\,\frac{1}{(D_0 + i\,\epsilon)\,(D_1 + i\,\epsilon)\,(D_3 + i\,\epsilon)} \\
I_{023} &= \int \frac{d^n k}{(2\,\pi)^n}\,\frac{1}{(D_0 + i\,\epsilon)\,(D_2 + i\,\epsilon)\,(D_3 + i\,\epsilon)}
\end{aligned}
$$

three 2-points integrals:

$$
\begin{aligned}
I_{03} &= \int \frac{d^n k}{(2\,\pi)^n}\,\frac{1}{(D_0 + i\,\epsilon)\,(D_3 + i\,\epsilon)} \\
I_{13} &= \int \frac{d^n k}{(2\,\pi)^n}\,\frac{1}{(D_1 + i\,\epsilon)\,(D_3 + i\,\epsilon)} \\
I_{23} &= \int \frac{d^n k}{(2\,\pi)^n}\,\frac{1}{(D_2 + i\,\epsilon)\,(D_3 + i\,\epsilon)}
\end{aligned}
$$

and four 1-point integrals:

$$
\begin{aligned}
I_0 &= \int \frac{d^n k}{(2\,\pi)^n}\,\frac{1}{(D_0 + i\,\epsilon)} \qquad & I_1 &= \int \frac{d^n k}{(2\,\pi)^n}\,\frac{1}{(D_1 + i\,\epsilon)} \\
I_2 &= \int \frac{d^n k}{(2\,\pi)^n}\,\frac{1}{(D_2 + i\,\epsilon)} \qquad & I_3 &= \int \frac{d^n k}{(2\,\pi)^n}\,\frac{1}{(D_3 + i\,\epsilon)}\ .
\end{aligned}
$$

[16]For an on line documentation on `form` see `http://www.nikhef.nl/~form/maindir/documentation/reference/online/`

[17]The code for the evaluation of A, B and C is found at `https://lectures.lapth.cnrs.fr/standard_model/cours/hgaga.frm`

Note that the last two sets of integrals would be ultraviolet divergent in 4 dimensions, but, working in n dimensions, they are regular and we can do translations on the loop momentum to evaluate them. For example:

$$I_1 = \int \frac{d^n k}{(2\,\pi)^n} \frac{1}{(k+p_1)^2 - M_W^2 + i\,\epsilon} = \int \frac{d^n k'}{(2\,\pi)^n} \frac{1}{(k')^2 - M_W^2 + i\,\epsilon} \quad \text{with} \quad k' = k + p_1 \; , \qquad (10.22)$$

then $I_1 = I_0$. In the same way one shows that:

$$I_3 = I_2 = I_1 = I_0$$

All 2-point integrals can be written in the following form:

$$J_2 = \int \frac{d^n k}{(2\,\pi)^n} \frac{1}{(k^2 - M_W^2 + i\,\epsilon)\,((k+p)^2 - M_W^2 + i\,\epsilon)} \; , \qquad (10.23)$$

is reduced to:

$$J_2 = \int_0^1 dx \int \frac{d^n k}{(2\,\pi)^n} \frac{1}{((k+p\,x)^2 - R^2 + i\,\epsilon)^2} \; , \qquad (10.24)$$

after introduction of the Feynman variable x, with $R^2 = M_W^2 - p^2\,x\,(1-x)$. Doing the change of variable k to $l = k + p\,x$ and using the usual formulae (see sec. 9.1) one obtains:

$$J_2 = \frac{i}{(4\,\pi)^{n/2}} \int_0^1 dx \, \left(R^2 - i\,\epsilon\right)^{-2+n/2} \frac{\Gamma(2-n/2)}{\Gamma(2)} \; . \qquad (10.25)$$

Introducing ε through $n = 4 - 2\,\varepsilon$, and expanding around $\varepsilon = 0$, it comes out:

$$J_2 = \frac{i}{(4\,\pi)^{2-\varepsilon}} \frac{\Gamma(1+\varepsilon)}{\varepsilon} \left(1 - \varepsilon \tilde{I}(p^2)\right) \; , \qquad (10.26)$$

with:

$$\tilde{I}(p^2) = \int_0^1 dx \, \ln\left(M_W^2 - p^2\,x\,(1-x) - i\,\epsilon\right) \; . \qquad (10.27)$$

The pole in ε is the consequence of the ultraviolet divergence of the 2-point functions. It turns out that, in our calculation, the 2-point integrals are all multiplied by ε which allows us to take the $\varepsilon \to 0$ limit to find finally:

$$\varepsilon\, J_2 = \frac{i}{(4\,\pi)^2}. \qquad (10.28)$$

For the 3-point integrals, both I_{013} et I_{023} can be written as:

$$J_3 = \int \frac{d^n k}{(2\,\pi)^n} \frac{1}{(k-r_1)^2 - M_W^2 + i\,\epsilon} \frac{1}{k^2 - M_W^2 + i\,\epsilon} \frac{1}{(k+r_2)^2 - M_W^2 + i\,\epsilon} \qquad (10.29)$$

with $r_1 = p_1$ and $r_2 = p_2$ for I_{013}, and $r_1 = p_2$ et $r_2 = p_1$ for I_{023}. Introducing the Feynman parameters and using $l = k + (r_2\,(1-x) - r_1\,x)\,y$ rather than k as integration variable one finds:

$$J_3 = 2 \int_0^1 y\,dy \int_0^1 dx \int \frac{d^n l}{(2\,\pi)^n} \frac{1}{(l^2 - R^2 + i\,\epsilon)^3} \; , \qquad (10.30)$$

with $R^2 = M_W^2 - y^2\, x\,(1-x)\, q^2$ and $q = p_1 + p_2 = r_1 + r_2$. Integrating on l yields:

$$J_3 = -\frac{i}{(4\,\pi)^{n/2}}\,\Gamma\left(3 - \frac{n}{2}\right)\int_0^1 y\, dy \int_0^1 dx\,(R^2 - i\,\epsilon)^{-3+n/2}\ , \tag{10.31}$$

which is regular. Taking $n = 4$ and doing the y integration J_3 can be written as:

$$J_3 = -\frac{i}{(4\,\pi)^2}\,\frac{1}{M_W^2}\, J\left(\frac{M_W^2}{q^2}\right)\ , \tag{10.32}$$

with the function J defined in eq. (9.16) of the previous section. The result depends only on $r_1 + r_2$ which implies $I_{013} = I_{023}$.
After contraction of the tensor $T^{\mu_1\,\mu_2}$ with the photons polarisation vectors we obtain:

$$\begin{aligned} T_W &= T^{\mu_1\,\mu_2}\,\varepsilon^*_{\mu_1}(p_1)\,\varepsilon^*_{\mu_2}(p_2) \\ &= \left(C\, g^{\mu_1\,\mu_2} + \frac{A}{p_1.p_2}\, p_1^{\mu_1}\, p_2^{\mu_2}\right)\,\varepsilon^*_{\mu_1}(p_1)\,\varepsilon^*_{\mu_2}(p_2) \end{aligned} \tag{10.33}$$

one observes that the B term has disappeared as it is multiplied by 0!. The `form` code gives $A = -C$ and:

$$C = \frac{i}{(4\,\pi)^2}\left(e^2\,\frac{e\,M_W}{\sin(\theta_W)}\right)\left[6 + \frac{1}{z_W} + J(z_W)\left(-12 + \frac{6}{z_W}\right)\right] \tag{10.34}$$

with $z_W = M_W^2/M_H^2$. Putting everything together it comes out:

$$\begin{aligned} T_W &= \frac{i}{(4\,\pi)^2}\left(e^2\,\frac{e\,M_W}{\sin(\theta_W)}\right)\left[6 + \frac{1}{z_W} + J(z_W)\left(-12 + \frac{6}{z_W}\right)\right]\left(g^{\mu_1\,\mu_2} - \frac{p_1^{\mu_1}\, p_2^{\mu_2}}{p_1.p_2}\right) \\ &\quad \times \varepsilon^*_{\mu_1}(p_1)\,\varepsilon^*_{\mu_2}(p_2) \\ &= i\,\frac{\alpha}{4\,\pi}\,\frac{e}{\sin(\theta_W)}\,\frac{M_H^2}{M_W}\,\mathcal{G}\left(\frac{M_W^2}{M_H^2}\right)\left(g^{\mu_1\,\mu_2} - \frac{2\,p_1^{\mu_1}\, p_2^{\mu_2}}{M_H^2}\right)\varepsilon^*_{\mu_1}(p_1)\,\varepsilon^*_{\mu_2}(p_2) \end{aligned} \tag{10.35}$$

where:

$$\mathcal{G}(z) = [6\,z + 1 + 6\,J\,(z)\,\,(1 - 2\,z)]$$

Some remarks are in order.

1. All ultraviolet divergences have disappeared: it was necessary to go to n dimensions in the intermediate steps of the calculation to give a mathematical meaning to individual integrals and allow for the momentum translations in the loops, but after combining all terms one takes the limit to 4 dimensions since the final result is regular;

2. $T^{\mu_1\,\mu_2}$ is transverse, which means $p_{1\,\mu_1}\, T^{\mu_1\,\mu_2} = p_{2\,\mu_2}\, T^{\mu_1\,\mu_2} = 0$.

10.4.2 Fermion loops

This part is very similar to the calculation of Higgs boson production via gluon-gluon fusion in sec. 9 and the result eq. (9.21) can be used with appropriate changes. First the strong coupling is replaced by $e\,Q_f$ and α_s then becomes $\alpha\,Q_f^2$ with $Q_f = -1, 2/3$ or $-1/3$. Since the photons are colour neutral the colour factor $Tr\left[T^a T^b\right] = \delta^{ab}/2$ becomes 1 (see eq. (9.6)). The result is:

$$T_f = -i\,\frac{\alpha\,Q_f^2\,e}{2\,\pi\,\sin\theta_W}\,\frac{M_H^2}{M_W}\,\mathcal{F}(z_f)\,\left(g^{\mu_1\,\mu_2} - \frac{2\,p_1^{\mu_1}\,p_2^{\mu_2}}{M_H^2}\right)\,\varepsilon^*_{\mu_1}(p_1)\,\varepsilon^*_{\mu_2}(p_2)\;, \tag{10.36}$$

with $z_f = m_f^2/M_H^2$. Eventhough heavy fermions only will contribute (Higgs coupling proportional to the fermion mass) it is necessary to recall that we have to sum over all fermions. The function $\mathcal{F}$ is defined in eq. (9.19) and we recall it here:

$$\mathcal{F}(z) = \left[2\,z + (1-4\,z)\;J\,(z)\right]\;.$$

The amplitude for the decay of a Higgs boson into two photons is then:

$$\begin{aligned} T_f \;&=\; -i\,\frac{\alpha\,e}{2\,\pi\,\sin\theta_W}\,\frac{M_H^2}{M_W}\,\left(g^{\mu_1\,\mu_2} - \frac{2\,p_1^{\mu_1}\,p_2^{\mu_2}}{M_H^2}\right)\,\varepsilon^*_{\mu_1}(p_1)\,\varepsilon^*_{\mu_2}(p_2) \\ &\times\left(\sum_l Q_l^2\,\mathcal{F}\left(\frac{m_l^2}{M_H^2}\right) + N\sum_q Q_q^2\,\mathcal{F}\left(\frac{m_q^2}{M_H^2}\right)\right) \end{aligned} \tag{10.37}$$

where the sum over l stands for leptons and q for quarks. In the latter case an extra factor N obviously appears from the colour sum in the loop.

10.5 Final result

The final amplitude will be the sum of the amplitudes T_f and T_W. To calculate its square one has to sum on the photon polarisation and evaluate the expression:

$$\begin{aligned} S \;&=\; \sum_{\lambda_1}\sum_{\lambda_2}\varepsilon^{\lambda_1 *}_{\mu_1}(p_1)\,\varepsilon^{\lambda_2 *}_{\mu_2}(p_2)\,\varepsilon^{\lambda_1}_{\nu_1}(p_1)\,\varepsilon^{\lambda_2}_{\nu_2}(p_2)\,\left(g^{\mu_1\,\mu_2} - \frac{2\,p_1^{\mu_2}\,p_2^{\mu_1}}{M_H^2}\right)\,\left(g^{\nu_1\,\nu_2} - \frac{2\,p_1^{\nu_2}\,p_2^{\nu_1}}{M_H^2}\right) \\ &=\; \left(g^{\mu_1\,\mu_2} - \frac{2\,p_1^{\mu_1}\,p_2^{\mu_2}}{M_H^2}\right)\,\left(g_{\mu_1\,\mu_2} - \frac{2\,p_{1\,\mu_1}\,p_{2\,\mu_2}}{M_H^2}\right) \\ &=\; 2 \end{aligned} \tag{10.38}$$

thus:

$$\overline{|T_W + T_f|^2} \;=\; \frac{\alpha^2\,e^2}{8\,\pi^2\,\sin\theta_W}\,\frac{M_H^4}{M_W^2}\,|Y|^2 \tag{10.39}$$

with:

$$\boxed{Y = \mathcal{G}\left(\frac{M_W^2}{M_H^2}\right) - 2\sum_l \mathcal{F}\left(\frac{m_l^2}{M_H^2}\right) - 2\,N\sum_q Q_q^2\,\mathcal{F}\left(\frac{m_q^2}{M_H^2}\right)}$$

Using eq. (10.3), the decay rate of a Higgs boson in two photons is:

$$\boxed{\Gamma_{H\to\gamma\gamma} = \frac{\alpha^3}{32\,\pi^2\,\sin^2(\theta_W)}\,\frac{M_H^3}{M_W^2}\,|Y|^2} \tag{10.40}$$