15 Majorana neutrinos

A Majorana fermion is a fermion which transforms into itself when applying the charge conjugation operator : a Majorana fermion is its own antiparticle. This is the case if we choose, in eq. (2.6), to identify the operators :

$$b(p) \equiv d(p), \qquad b^\dagger(p) \equiv d^\dagger(p) \tag{15.1}$$

so that:

$$\psi^M(x) = \int \frac{d^3p}{(2\pi)^3 2\omega} \left[b(p)\; u(p)\; e^{-ip.x} + b^\dagger(p)\; v(p)\; e^{ip.x} \right]. \tag{15.2}$$

In the case of a "left-handed" neutrino eq. (3.23),

$$\psi_L(x) = \int \frac{d^3p}{(2\pi)^3 2\omega} \left[b_L(p)\; u_L(p)\; e^{-ip.x} + b_R^\dagger(p)\; v_R(p)\; e^{ip.x} \right], \tag{15.3}$$

we have using eqs. (B.12) in the appendix, $u_L^c = v_l$, $v_R^c = u_R$,

$$\begin{aligned} \psi_L^c(x) &= \int \frac{d^3p}{(2\pi)^3 2\omega} \left[b_R(p)\; u_R(p)\; e^{-ip.x} + b_L^\dagger(p)\; v_L(p)\; e^{ip.x} \right] \\ &= \psi_R(x) \end{aligned} \tag{15.4}$$

and defining a Majorana neutrino by

$$\psi^M(x) = \psi_L(x) + \psi_L^c(x), \tag{15.5}$$

one has indeed,

$$\boxed{(\psi^M(x))^c = \psi^M(x),} \tag{15.6}$$

which satisfies the Majorana condition. A Majorana neutrino has both left-handed and a right-handed components which are related by the $\mathcal{C}$ transformation. A mass for a Majorana neutrino is generated very simply from a term in the lagrangian density such as

$$\bar{\psi}_L^c(x)\psi_L(x) + \bar{\psi}_L(x)\psi_L^c(x) \equiv \bar{\psi}^M(x)\psi^M(x). \tag{15.7}$$

without introduction of an extra right-handed component.

15.1 Majorana mass term for neutrinos

Coming back to the Standard Model with three generations, we recall that we have defined, eq. (12.1),

$$\boldsymbol{\nu'}_{\!L} = \begin{pmatrix} \nu_{e_L} \\ \nu_{\mu_L} \\ \nu_{\tau_L} \end{pmatrix},$$

a triplet of left-handed neutrino flavour eigenstates. A Yukawa mass term is defined by[100],

$$\mathcal{L}_{Y_M} = -\frac{1}{2}\,(\overline{\boldsymbol{\nu}'^c_L}\,\boldsymbol{M}\,\boldsymbol{\nu}'_L + \overline{\boldsymbol{\nu}'_L}\,\boldsymbol{M}^\dagger\,\boldsymbol{\nu}'^c_L) \tag{15.8}$$

where $\boldsymbol{M}$ is a 3×3 complex matrix. One shows first that this matrix is symmetric, $\boldsymbol{M} = \boldsymbol{M}^T$. Indeed, from eq. (B.11) in the appendix, and using $i\gamma_2^T = i\gamma_2$,

$$\begin{aligned} \overline{\boldsymbol{\nu}'^c_L}\,\boldsymbol{M}\,\boldsymbol{\nu}'_L &= \boldsymbol{\nu}'^T_L\, i\gamma_2\gamma_0\,\boldsymbol{M}\,\boldsymbol{\nu}'_L \\ &= -\,\boldsymbol{\nu}'^T_L\,\boldsymbol{M}^T\,\gamma_0 i\gamma_2\,\boldsymbol{\nu}'_L \;=\; \boldsymbol{\nu}'^T_L\, i\,\gamma_2\gamma_0\,\boldsymbol{M}^T\,\boldsymbol{\nu}'_L \end{aligned} \tag{15.9}$$

where the second line is obtained from the first by transposition with a change of sign due to the anticommutation of fermions, which proves the symmetry property of the mass matrix. The complex matrix $\boldsymbol{M}$ is diagonalised by the matrix $\boldsymbol{S}$

$$\boldsymbol{M} = \boldsymbol{S}^T\,\boldsymbol{m}\;\boldsymbol{S}, \tag{15.10}$$

with $\boldsymbol{m}$ is diagonal with real eigenvalues. Plugging this expression in $\mathcal{L}_{Y_M}$, the mass term is written

$$\begin{aligned} \mathcal{L}_{Y_M} &= -\frac{1}{2}\,(\overline{\boldsymbol{\nu}'^c_L}\,\boldsymbol{M}\,\boldsymbol{\nu}'_L + \overline{\boldsymbol{\nu}'_L}\,\boldsymbol{M}^\dagger\,\boldsymbol{\nu}'^c_L) = -\frac{1}{2}\,(\boldsymbol{\nu}'^T_L\, i\gamma_2\gamma_0\,\boldsymbol{S}^T\,\boldsymbol{m}\;\boldsymbol{S}\,\boldsymbol{\nu}'_L + \boldsymbol{\nu}'^\dagger_L\gamma_0\,\boldsymbol{S}^\dagger\,\boldsymbol{m}\;\boldsymbol{S}^* i\gamma_2\boldsymbol{\nu}'^*_L) \\ &= -\frac{1}{2}\,(\overline{\boldsymbol{\nu}^c_L}\,m\,\boldsymbol{\nu}_L + \overline{\boldsymbol{\nu}_L}\,m\,\boldsymbol{\nu}^c_L), \end{aligned} \tag{15.11}$$

where we have defined

$$\boldsymbol{\nu}_L = \boldsymbol{S}\,\boldsymbol{\nu}'_L \quad\Leftrightarrow\quad \boldsymbol{\nu}^*_L = \boldsymbol{S}^*\,\boldsymbol{\nu}'^*_L \quad\Leftrightarrow\quad \boldsymbol{\nu}^\dagger_L = \boldsymbol{\nu}'^\dagger_L\boldsymbol{S}^\dagger. \tag{15.12}$$

The Yukawa mass term can then be simplified to

$$\mathcal{L}_{Y_M} = -\frac{1}{2}\,\overline{\boldsymbol{\nu}}\,m\,\boldsymbol{\nu} = -\frac{1}{2}\sum_i m_i\overline{\nu_i}\nu_i \tag{15.13}$$

with $\boldsymbol{\nu} = \boldsymbol{\nu}_L + \boldsymbol{\nu}^c_L$, a triplet of Majorana neutrinos diagonalising the mass term. This shows that one could, in principle, give a mass to neutrinos solely from left-handed neutrinos.

To generate in the Standard Model a mass term by spontaneous symmetry breaking, coupling a left-handed neutrino to its conjugate $\boldsymbol{\nu}^c_L$ and a $SU(2)$ scalar doublet field Φ, we turn to sec. 8.3. Hovever a possible candidate like $\overline{\psi}_{e_L}\tilde{\Phi}\,\nu^c_L$ is not acceptable since is not a singlet under $SU(2)_L\otimes U(1)_Y$: $\overline{\psi}_{e_L}\tilde{\Phi}$ is indeed a singlet but ν_L is a $SU(2)_L$ doublet with a non-vanishing y quantum number. One could introduce a more complicated stucture[101],

$$\mathcal{L}_{Y_M} = c\,\overline{\psi}_{e_L}\tilde{\Phi}\,(\tilde{\Phi}\overline{\psi}_{e_L})^c, \tag{15.14}$$

[100] Note the factor 1/2, compared to a Dirac mass term, because $\boldsymbol{\nu}'$ and $\overline{\boldsymbol{\nu}'}$ contain the same degrees of freedom.
[101] S. Weinberg, Phys. Rev. Lett. **43** (1979) 1566.

which after symmetry breaking gives a mass to neutrinos of order $m_\nu \simeq c\ v^2 \simeq c\ G_F^{-1}$. Since this term has dimension 5, the coupling c is necessarily of the form $1/\Lambda$, with Λ a large scale introduced to keep the neutrinos very light. But this interaction is not renormalizable and it would require the introduction of new particles to render the theory finite in analogy with what is done to go from the Fermi model to the Standard Model. Therefore, it seems difficult to generate massive neutrinos without introducing new degrees of freedom.

15.2 Neutrino masses and the see-saw mechanism

We restrict the discussion to a one generation model and postulate a very massive right-handed neutrino singlet, N_R, under the gauge group (sterile neutrino since non-interacting with gauge bosons). The Yukawa lagrangian is assumed to have both a Dirac mass term (arising from the usual symmetry breaking mechanism with a scalar field doublet) and a Majorana mass term, coupling the right-handed neutrino to its charge conjugate, of the following form,

$$\begin{aligned}\mathcal{L}_Y &= -m_D \overline{N_R}\, \nu_L - \frac{1}{2} M_R\, \overline{N_R}\, N_R^c + \text{ h.c.} \\ &= -\frac{1}{2}(\overline{\nu_L^c}\, \overline{N_R}) \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ N_R^c \end{pmatrix} + \text{ h.c.,}\end{aligned} \tag{15.15}$$

where we have used $\overline{\nu_L^c}\, N_R^c = \overline{N_R}\, \nu_L$ to recover the first line from the matrix expression of the second one. We assume $m_D \ll M_R$, m_D being of the order of the electroweak symmetry breaking scale and M_R much larger (of the order of a grand unification scale?). The symmetric mass matrix can be diagonalised and, taking into account the hierarchy of the two mass scales, one finds eigenvalues approximately equal to $-m_D^2/M_R$ and M_R. To make both eigenvalues positive we rather write

$$\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \approx \begin{pmatrix} i & \rho \\ -i\rho & 1 \end{pmatrix} \begin{pmatrix} \rho^2 M_R & 0 \\ 0 & M_R \end{pmatrix} \begin{pmatrix} i & -i\rho \\ \rho & 1 \end{pmatrix} \tag{15.16}$$

with $\rho = m_D/M_R \ll 1$, so that the eigenstates of the mass matrix are

$$\begin{aligned}\nu_{1L} &= i(\nu_L - \rho N_R^c) \\ N_{1L} &= \rho\nu_L + N_R^c \simeq N_R^c,\end{aligned} \tag{15.17}$$

and the Yukawa term can be written

$$\begin{aligned}\mathcal{L}_Y &= -\frac{1}{2}(\overline{\nu_{1L}^c}\, \overline{N_{1L}}) \begin{pmatrix} \rho^2 M_R & 0 \\ 0 & M_R \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ N_{1L}^c \end{pmatrix} + \text{ h.c.} \\ &= -\ \frac{1}{2}\, \rho^2 M_R\, \overline{\nu_1}\, \nu_1^c - \frac{1}{2} M_R\, \overline{N_1}\, N_1^c,\end{aligned} \tag{15.18}$$

after introducing the Majorana neutrinos

$$\nu_1 = \nu_{1L} + \nu_{1L}^c, \qquad N_1 = N_{1L} + N_{1L}^c. \tag{15.19}$$

To summarize, from a light left-handed "Dirac" neutrino and a heavy right-handed "Majorana" neutrino, the symmetric mass matrix of type eq. (15.15), can be diagonalised leading to two Majorana neutrinos, a light one, ν_1 with mass $\rho^2 M_R$, and a heavy one with mass M_R. The light left-handed neutrino ν_{1L} has a small mixing component with the heavy neutrino which could in principle be produced at the LHC, if its mass is not too high.

The procedure above can be generalised to the three generations of the Standard Model. One of the simplest way (type I see-saw) is to introduce three right-handed heavy neutrinos, N_{R_i}, singlets under the gauge group, similar to what is done for charged leptons, and add in the Yukawa Lagrangian, besides the term coupling to the scalar doublet field Φ, a Majorana mass term for the N_{R_i}'s:

$$\mathcal{L}_Y = -\frac{1}{2}\sum_{i=1,2,3} M_{Ri}\, \overline{N_{Ri}^c}\, N_{Ri} - \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} c_{\alpha i}\overline{\psi}_{\alpha_L}\tilde{\Phi} N_{Ri} + \text{ h.c. } . \tag{15.20}$$

The second term $c_{\alpha i}(\overline{\nu_L}\Phi^0 - \overline{e_L}\Phi^-)N_{Ri}$, after symmetry breaking, generates Dirac mass parameters $m_{D\alpha i} = c_{\alpha i} v/\sqrt{2}$. The Yukawa term can be written in a matrix form identical to eq. (15.15) where

$$(\nu_L^T\ N_R^{cT}) = (\nu_{Le}^T\ \nu_{L\mu}^T\ \nu_{L\tau}^T\ N_{R1}^{cT}\ N_{R2}^{cT}\ N_{R3}^{cT}) \tag{15.21}$$

is the transpose of a six-component spinor and the mass matrix

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \tag{15.22}$$

is a $6 \otimes 6$ matrix contructed from the $3 \otimes 3$ matrix m_D with elements $m_{D\alpha i}$ and M_R a $3 \otimes 3$ diagonal matrix with elements M_{Ri}. This symmetric matrix $\mathcal{M}$ can be diagonalised yielding 3 light eigenvalues of order

$$m_\nu \simeq -m_D\, M_R^{-1} m_D^T \tag{15.23}$$

and 3 heavy ones. The associated eigenstates are Majorana neutrinos. Introducing these mass eigenstates in the charged current interactions term will yield a **PMNS** mixing matrix exactly as before, in the case of Dirac neutrinos. There is a difference however since it is not allowed to rotate away the phases of the neutrino fields[102]: the phase of a Majorana neutrino is fixed by the condition

[102] J. Bernabeu, P. Pascual, Nucl. Phys. **B228** (1983) 21; S.T. Petcov, Adv. High Energy Phys. 2013 (2013) 852987, arXiv:1303.5819 [hep-ph] .

$\nu_i^c = \mathcal{C}\gamma_0\nu_i^* = \nu_i$. In the expression of the charged current lagrangian, eq. (12.8), only the phases of the charged lepton fields can be changed, $e_{\alpha_L} \to e^{-i\phi_{e_\alpha}} e_{\alpha_L}$, so that the **PMNS** matrix elements $(\mathbf{S}_\nu^\dagger)_{\alpha j}$ become $e^{-i\phi_{e_\alpha}}(\mathbf{S}_\nu^\dagger)_{\alpha j}$ (see the discussion after eq. (11.11)). These three arbitrary phases are used to absorb three phases of the **PMNS** matrix, which can be written in the form, see eq. (11.12),

$$\mathbf{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}e^{i\delta}s_{23} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}, \qquad (15.24)$$

with three angles and three phases: the $\mathcal{CP}$ phase δ and the 2 Majorana phases $0 < \alpha_1 < 2\pi$ and $0 < \alpha_2 < 2\pi$. The pattern of oscillations of Majorana neutrinos is the same as that of Dirac neutrinos since the combination which controls the change of flavour (see eq. (12.9)),

$$U_{\alpha i}U^*_{\alpha j}U^*_{\beta i}U_{\beta j},$$

in eq. (12.17) is independent of the form of the **PMNS** matrix, eqs. (11.12) or (15.24), and it is not possible from the study of oscillations to distinguish Majorana from Dirac neutrinos.

If one considers a global phase change on all left-handed fields

$$\nu'_{i_L}(x) = e^{i\Lambda}\,\nu_{i_L}(x) \quad \Leftrightarrow \quad \chi'_{i_L}(x) = e^{i\Lambda}\,\chi_{i_L}(x); \qquad l'(x) = e^{i\Lambda}\,l(x), \qquad (15.25)$$

the gauge interaction part remains invariant but this is not the case for the Yukawa term in the lagrangian since the right-handed fields are not independent and one has

$$\overline{\boldsymbol{\nu}'^c_L}(x) = e^{i\Lambda}\,\overline{\boldsymbol{\nu}^c_L}(x). \qquad (15.26)$$

Invariance under the global phase change eq. (15.25) is associated to lepton number conservation $L = L_e + L_\nu + L_\tau$. In the presence of a Yukawa mass term, the invariance is lost and the lepton number is not conserved: it is possible to have nuclear transitions with emission of two electrons without neutrino

$$(A, Z) \to (A, Z+2) + e^- + e^-, \qquad (15.27)$$

i.e. a neutrinoless double-beta decay $\{0\nu\beta\beta\}$. This is illustrated in fig. 27. In a first beta decay in a nucleus, a $\overline{\nu}_e$ is produced which turns into a ν_e via the Majorana Yukawa mass term eq. (15.8) followed by the reaction $\nu_e + n \to e^- + p$. The rate of transition is minute : proportional the $G_F^4\, m_\nu^2$ the fourth power of Fermi constant and the square of the neutrino mass ! Several experiments (CUORE[103],

[103]CUORE Collaboration, K. Alfonso *et al.*, Phys. Rev. Lett. **115** (2015) 102502.

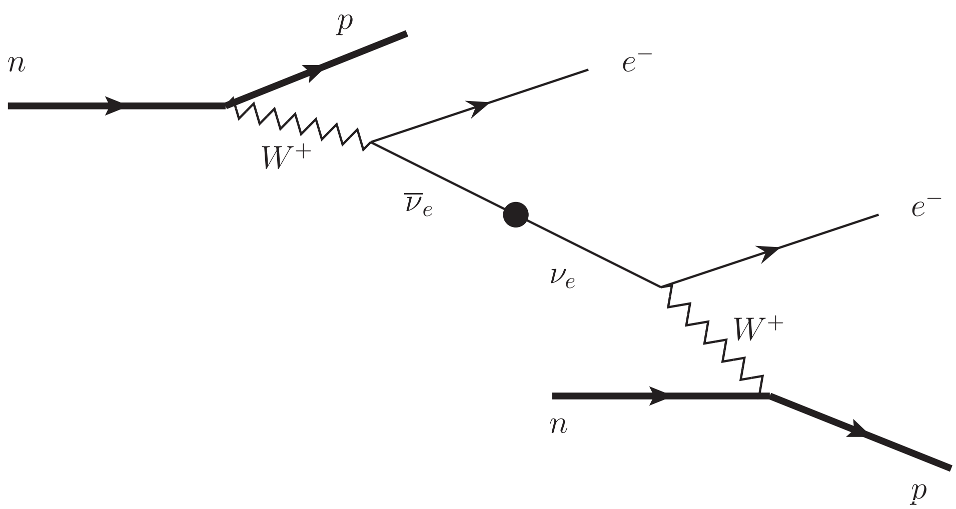

Figure 27: $\{0\nu\beta\beta\}$*decay: neutrinoless double-beta decay.*

NEMO[104], KamLAND-Zen[105], SNO+[106], SuperNEMO[107]) have searched, are searching or will search for this process which is essentially the unique signature of the Majorana nature of the neutrinos[108].

[104]NEMO Collaboration, R. Arnold *et al.*, Phys. Rev. **D89** (2014) 111101.
[105]KamLAND-Zen Collaboration, A. Gando *et al.*, Phys. Rev. Lett. **110** (2013) 062502.
[106]SNO+ Collaboration, S. Andringa, *et al.*, Adv. High Energy Phys. **2016** (2016) 6194250, arXiv:1508.05759 [physics.ins-det].
[107]NEMO-3 Collaboration, R. Arnold *et al.*, Phys. Rev. **D92** (2015) 072011.
[108]For a review on neutrinoless double β decay, see Stefano Dell'Oro *et al.*, Adv. High Energy Phys. **2016** (2016) 2162659, arXiv:1601.07512 [hep-ph].