2 The Fermi theory and its extensions

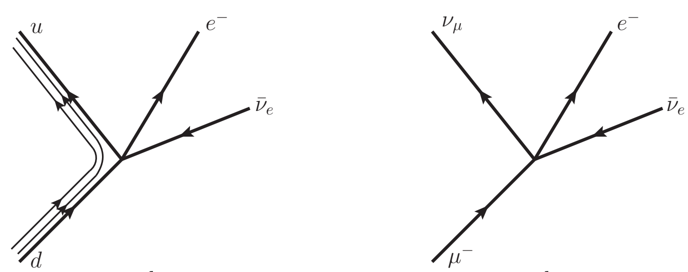
At the beginning was the Fermi theory of muon decay :

$$\mu^- \to e^- \ \overline{\nu}_e \ \nu_\mu.$$

and neutron decay :

$$n \to p \ e^- \ \overline{\nu}_e.$$

In the latter case we work in the quark/parton model where we assume the nucleon is made of three quarks : neutron = (udd) and proton = (uud). For neutron decay, charge conservation allows only the transition $d \to u \ e^- \ \overline{\nu}_e$, the other two quarks being spectators.



neutron decay *μ⁻decay*

2.1 Contact interactions

These transitions are described by a local current-current (4 fermion) interaction parameterised by the Lagrangian:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} J^\nu(x) J_\nu^\dagger(x). \tag{2.1}$$

The current has a leptonic part and a hadronic part, $J_\nu(x) = l_\nu(x) + h_\nu(x)$,

$$\begin{aligned} l_\nu(x) &= \overline{\psi}_e \gamma_\nu (1-\gamma_5)\psi_{\nu_e} + \overline{\psi}_\mu \gamma_\nu (1-\gamma_5)\psi_{\nu_\mu} + \overline{\psi}_\tau \gamma_\nu (1-\gamma_5)\psi_{\nu_\tau} \\ h_\nu(x) &= \overline{\psi}_d \gamma_\nu (1-\gamma_5)\psi_u + \overline{\psi}_s \gamma_\nu (1-\gamma_5)\psi_c + \overline{\psi}_b \gamma_\nu (1-\gamma_5)\psi_t, \end{aligned} \tag{2.2}$$

where, for simplicity, the argument of the fermion fields are not shown, ψ_e instead $\psi_e(x)$, $\cdots$. The γ_5 matrix anticommutes with all γ_ν's (see the appendix for the properties of the γ_5 matrix). The particular $V - A$ (vector (γ_ν) $-$ axial ($\gamma_\nu\gamma_5$)) form of the current is dictated by experiment, in particular the angular distribution of particles in the final state[1]. The Fermi constant G_F is universal,

[1]The $\gamma_\nu\gamma_5$ interaction breaks parity maximally, see sec.B.2 in the appendix.

i.e. it is the same for the hadronic sector and the leptonic sector and its value has been measured to be :

$$G_F = 1.6639(2)10^{-5} \text{ GeV}^{-2}. \tag{2.3}$$

Thus the transition matrix element for μ decay is an element in $(G/\sqrt{2})l^\nu(x)l^\dagger_\nu(x)$ constructed from the first two terms of $l^\nu(x)$:

$$\begin{aligned}\mathcal{M} &= \frac{G_F}{\sqrt{2}}(\overline{\psi}_e\gamma_\nu(1-\gamma_5)\psi_{\nu_e})(\overline{\psi}_\mu\gamma^\nu(1-\gamma_5)\psi_{\nu_\mu})^\dagger \\ &= \frac{G_F}{\sqrt{2}}(\overline{\psi}_e\gamma_\nu(1-\gamma_5)\psi_{\nu_e})(\overline{\psi}_{\nu_\mu}\gamma^\nu(1-\gamma_5)\psi_\mu)\end{aligned} \tag{2.4}$$

and that of neutron decay (d quark decay) is an element of $(G/\sqrt{2})l^\nu(x)h^\dagger_\nu(x)$:

$$\begin{aligned}\mathcal{M} &= \frac{G_F}{\sqrt{2}}(\overline{\psi}_e\gamma_\nu(1-\gamma_5)\psi_{\nu_e})(\overline{\psi}_d\gamma^\nu(1-\gamma_5)\psi_u)^\dagger \\ &= \frac{G_F}{\sqrt{2}}(\overline{\psi}_e\gamma_\nu(1-\gamma_5)\psi_{\nu_e})(\overline{\psi}_u\gamma^\nu(1-\gamma_5)\psi_d).\end{aligned} \tag{2.5}$$

Introducing the expansion of a spinor ψ_i in terms of plane waves with annihilation operators $b_i^{(\alpha)}(p)$ and $d_i^{(\alpha)}(p)$ for a positive energy and negative energy particle respectively (α is the polarisation index):

$$\begin{aligned}\psi_i(x) &= \int \frac{d^3p}{(2\pi)^3 2\omega}\ \psi_i(p,x) \\ &= \int \frac{d^3p}{(2\pi)^3 2\omega}\sum_\alpha \left[b_i^{(\alpha)}(p)\ u_{i\alpha}(p)\ e^{-ip.x} + d_i^{(\alpha)\dagger}(p)\ v_{i\alpha}(p)\ e^{ip.x}\right], \qquad p.x = \omega t - \mathbf{p}.\mathbf{x},\end{aligned} \tag{2.6}$$

where the $u_{i\alpha}(p)$ and $v_{i\alpha}(p)$ are, respectively, the wave functions of the annihilated fermion (positive energy) and the created antifermion (negative energy). Injecting eq. (2.6) into the matrix element above, we see that eq. (2.5) describes several processes related by crossing symmetry such as: $d \to u\ e^-\ \bar{\nu}_e$ (term in $\bar{u}_u \cdots u_d\ \bar{u}_e \cdots v_{\nu_e}$) or $d\ \bar{u} \to e^-\ \bar{\nu}_e$ (term in $\bar{v}_u \cdots u_d\ \bar{u}_e \cdots v_{\nu_e}$) or $\nu_e\ d \to e^-\ u$ (term in $\bar{u}_u \cdots u_d\ \bar{u}_e \cdots u_{\nu_e}$) or $\cdots$. Considering the last process which is the dominant mechanism for the deep inelastic scattering of a neutrino on a proton one can easily calculate the cross section at the partonic level. Defining the momenta by $\nu_e(p_1)\ d(p_2) \to e^-(p_3)\ u(p_4)$, the invariants are

$$(p_1+p_2)^2 = s, \qquad (p_1-p_3)^2 = t = q^2 = (s/2)(1-\cos\theta), \qquad (p_1-p_4)^2 = u = (s/2)(1+\cos\theta). \tag{2.7}$$

Supposing all fermions massless, the matrix element is (ignoring the polarisation indices):

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\ [\bar{u}_e(p_3)\gamma^\mu(1-\gamma_5)u_{\nu_e}(p_1)]\ [\bar{u}_u(p_4)\gamma_\mu(1-\gamma_5)u_d(p_2)], \tag{2.8}$$

and the matrix element squared summed/averaged over polarisation is

$$\overline{\Sigma}|\mathcal{M}|^2 = \frac{1}{4}\frac{G_F^2}{2}\, 4\, \mathrm{Tr}(\not p_3\gamma^\mu \not p_1\gamma^\nu(1-\gamma_5))\mathrm{Tr}(\not p_4\gamma_\mu \not p_2\gamma_\nu(1-\gamma_5))$$
$$= \frac{G_F^2}{2}[\mathrm{Tr}(\not p_3\gamma^\mu \not p_1\gamma^\nu)\mathrm{Tr}(\not p_4\gamma_\mu \not p_2\gamma_\nu) + \mathrm{Tr}(\not p_3\gamma^\mu \not p_1\gamma^\nu\gamma_5)\mathrm{Tr}(\not p_4\gamma_\mu \not p_2\gamma_\nu\gamma_5)]. \tag{2.9}$$

The traces and their product can be easily evaluated using eqs. (A.9) and (A.10) in appendix A. There is no mixing between the trace with a γ_5 matrix and that without since the former is antisymmetric in $\mu\nu$ and the latter is symmetric. After reduction the result is simple:

$$\overline{\Sigma}|\mathcal{M}|^2 = 4\, G_F^2\, [(s^2+u^2) \ + \ (s^2-u^2)] = 8\, G_F^2\, s^2, \tag{2.10}$$

where the first term in the square brackets corresponds to the first term ($V-A$ interaction) in eq. (2.9). The differential cross section is[2]:

$$\frac{d\sigma}{d\Omega}^{\nu_e d\to e^- u} = \frac{1}{2s}\int \frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^{(4)}(p_1+p_2-p_3-p_4)\left(\overline{\Sigma}|\mathcal{M}|^2\right)$$
$$= \left\{\frac{1}{(2\pi)^2}\frac{1}{16s}\right\}(8\, G_F^2\, s^2)$$
$$= \frac{G_F^2}{8\pi^2}\, s, \tag{2.11}$$

independent of the polar angle. This result is in agreement with the data at not too high s. It is also interesting to consider in the Fermi model the diffusion of antineutrinos on the proton. In the quark/parton model, because of charge conservation, the $\bar{\nu}_e$ interacts only with the u quarks via the transition $\bar{\nu}_e(p_1)\, u(p_2) \to e^+(p_3)\, d(p_4)$. The matrix element can easily be constructed and it is:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}}\, [\bar{v}_{\nu_e}(p_1)\gamma^\mu(1-\gamma_5)v_e(p_3)]\ [\bar{u}_d(p_4)\gamma_\mu(1-\gamma_5)u_u(p_2)]. \tag{2.12}$$

Taking the square of the matrix element one obtains eq. (2.9) with p_1 and p_3 interchanged. Because $\mathrm{Tr}(\not p_3\gamma^\mu \not p_1\gamma^\nu)$ is symmetric and $\mathrm{Tr}(\not p_3\gamma^\mu \not p_1\gamma^\nu\gamma_5)$ antisymmetric under this interchange, one sees immediately that $\overline{\Sigma}|\mathcal{M}|^2 = 4\, G_F^2\, [(s^2+u^2)-(s^2-u^2)] = 8\, G_F^2\, u^2$ and consequently the differential cross section is found to be:

$$\frac{d\sigma}{d\Omega}^{\bar{\nu}_e u\to e^+ d} = \frac{G_F^2}{8\pi^2}\,\frac{u^2}{s} = \frac{G_F^2}{8\pi^2}\,\frac{s}{4}\,(1+\cos\theta)^2 \tag{2.13}$$

also in agreement with experimental observations where the positron is produced mainly in the direction of the initial $\bar{\nu}_e$ quark.

[2] The term in $\{\cdots\}$ is the phase space factor for massless particles.

If instead of the $V - A$ form of the currents we had used only the vector part the results would be in disagreement with data since for both cross sections above, eqs (2.11) and (2.13), the result would have been :

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2}{64\pi^2} \frac{s^2+u^2}{s}, \tag{2.14}$$

a prediction not supported by experiments because of the wrong angular distribution for both reactions. If one had tried "scalar currents" of the form:

$$\begin{aligned} l(x) &= \overline{\psi}_e \psi_{\nu e} + \overline{\psi}_\mu \psi_{\nu_\mu} + \cdots \\ h(x) &= \overline{\psi}_d \psi_u + \overline{\psi}_s \psi_c + \cdots \,,, \end{aligned} \tag{2.15}$$

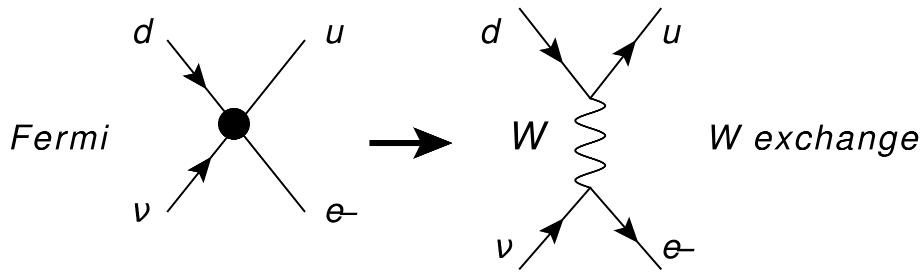
both ν and $\bar{\nu}$ cross sections would have been proportional to s : this prediction is correct for $\nu_e\, d \to e^-\, u$ but incorrect for $\bar{\nu}_e\, u \to e^+\, d$. In summary, all low energy data support the V − A form to describe weak interactions.

However the Fermi theory is not satisfactory at high energy. Indeed, from eq. (2.11) one obtains for the total cross section $\sigma^{\nu_e d \to e^- u} = G_F^2\, s/2\pi$. However such a rapid rise of the cross section with energy cannot be asymptotically true as it violates the famous Froissart unitarity bound which requires $\sigma \leq \ln^2 s$ as $s \to \infty$. Note that the linear rise in s of a $2 \to 2$ cross section integrated over all final state variables could have easily been guessed on dimensional grounds. Indeed, in Fermi theory, such a cross section is proportional to G_F^2 of dimension GeV^{-4} but a cross section[3] is measured in units of GeV^{-2}. Since, after integrating over the final state phase space, the only scale available in the problem is s, of dimension GeV^2, one necessarily has $\sigma \propto G_F^2\, s$.

2.2 Vector boson mediated interactions

The rapid rise of cross sections is related to the locality of the current-current interaction. One can make the 4-fermion interaction nonlocal by postulating a massive charged particle coupling to the $J_\mu(x)$ current similarly to the coupling of a photon to the fermionic current $\bar{\psi}(x)\gamma_\mu\psi(x)$ to mediate the interaction between the currents. It must be a vector particle because of the γ_μ coupling in eq. (2.2) as shown below

[3] We work in the system where $\hbar = c = 1$.



Denoting M_W the mass of this particle and g_W its dimensionless coupling to the currents, the matrix element eq. (2.5) becomes :

$$\mathcal{M} = g_W^2 [\overline{\psi}_e \gamma_\mu (1-\gamma_5)\psi_{\nu_e}] \frac{g^{\mu\nu} - q^\mu q^\nu / M_W^2}{q^2 - M_W^2} [\overline{\psi}_u \gamma_\nu (1-\gamma_5)\psi_d] \tag{2.16}$$

where q is the momentum transfer from the d quark to the u quark. Coming back to the reaction $\nu_e(p_1)\, d(p_2) \to e(p_3)\, u(p_4)$ studied above, the matrix element eq. (2.8), in momentum space is (we do not write explicitely the polarisation index of the fermions):

$$\begin{aligned} \mathcal{M} &= g_W^2 [\bar{u}_e(p_3)\gamma_\mu(1-\gamma_5)u_{\nu_e}(p_1)] \frac{g^{\mu\nu} - q^\mu q^\nu / M_W^2}{q^2 - M_W^2} [\bar{u}_u(p_4)\gamma_\nu(1-\gamma_5)u_d(p_2)] \\ &= \frac{g_W^2}{q^2 - M_W^2}\, [\bar{u}_e(p_3)\gamma_\mu(1-\gamma_5)u_{\nu_e}(p_1)]\ [\bar{u}_u(p_4)\gamma^\mu(1-\gamma_5)u_d(p_2)], \end{aligned} \tag{2.17}$$

with $q = p_1 - p_3 = p_4 - p_2$ and where we have used Dirac equation for massless fields $\not{p}_i u(p_i) = 0$. This equation is identical to eq (2.8) provided we make the substitution:

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{q^2 - M_W^2} \to \frac{g_W^2}{M_W^2} \quad \text{when } q^2 \to 0, \tag{2.18}$$

which allows to obtain the matrix element squared summed/averaged over polarisation from eq. (2.8):

$$\overline{\Sigma}|\mathcal{M}|^2 = 16\ g_W^4\ \frac{s^2}{(q^2 - M_W^2)^2}, \tag{2.19}$$

and the differential cross section :

$$\frac{d\sigma}{d\Omega}^{\nu_e d \to e^- u} = \frac{g_W^4}{4\pi^2}\ \frac{s}{(q^2 - M_W^2)^2}, \tag{2.20}$$

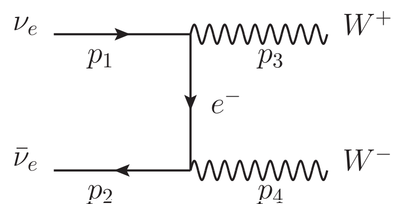
with $q^2 = -s(1-\cos\theta)/2$. The integrated cross section is easily calculated to be:

$$\sigma^{\nu_e d \to e^- u} = \frac{1}{\pi} \frac{g_W^4}{M_W^2} \frac{s}{s + M_W^2}. \tag{2.21}$$

At low energy we recover the Fermi model prediction provided, $g_W^2/M_W^2 = G_F\sqrt{2}$, while at high energy the Froissart bound is satisfied.

2.3 Still more problems!

However this is not the end of the story ! The W particle can be produced, and has been produced at LEP2, in the reaction $e^- \ e^+ \to W^- \ W^+$ but the corresponding cross section, in our model, violates Froissart bound. To see this, let us consider instead the unrealistic, but simpler, case[4] of the scattering $\nu_e(p_1)\ \bar{\nu}_e(p_2) \to W^+(p_3)\ W^-(p_4)$ the amplitude of which is given by only one Feynman diagram with the exchange of an electron:



To illustrate the problem we first define the kinematics and make some comments on the polarisation states of a massive vector particle. We work at very high energy in the center of mass frame of the $e^- \ e^+$ system :

$$(p_1+p_2)^\mu = (\sqrt{s}, \mathbf{0}), \quad p_1^\mu = (\frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2}), \quad p_2^\mu = (\frac{\sqrt{s}}{2}, 0, 0, -\frac{\sqrt{s}}{2}). \tag{2.22}$$

We take p_3 and p_4 in the xOz plane:

$$p_3^\mu = (E_3, p_3 \sin\theta, 0, p_3 \cos\theta), \qquad E_3 = \sqrt{s}/2, \qquad p_3 = \sqrt{s/4 - M_W^2} \tag{2.23}$$

Unlike the photon which has two transverse polarisation states the W particle being massive has three degrees of polarisation.

• Polarisation of a massive spin 1 particle

In the rest frame of a massive particle, $p = (M, \mathbf{0})$ the polarisation is described by space-like vectors. A basis of such vectors is given by

$$\varepsilon^{(1)\mu} = (0,1,0,0), \quad \varepsilon^{(2)\mu} = (0,0,1,0), \quad \varepsilon^{(3)\mu} = (0,0,0,1), \tag{2.24}$$

satisfying $\varepsilon^{(i)}.\varepsilon^{(j)} = -\delta^{ij}$ as well as $p.\varepsilon^{(i)} = 0$ for $i, j = 1, 2,$ or 3.

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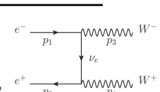
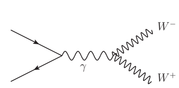
[4]For $e^- \ e^+$ there are two diagrams   , and this complicates the discussion.

Important remark

For a boson W with momentum p, the polarisation vectors become functions of p, $\varepsilon^{(i)\mu}(p)$, satisfying the same conditions as above, namely $\varepsilon^{(i)}(p).\varepsilon^{(j)}(p) = -\delta^{ij}$ as well as $p.\varepsilon^{(i)}(p) = 0$. One often needs, in the propagator for example,

$$\mathcal{P}^{\mu\nu} = \sum_i \varepsilon^{(i)\mu}(p)\, \varepsilon^{(i)\nu}(p) = -\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{M_W^2} \right). \tag{2.25}$$

The last equality is easily derived knowing that the rank 2 tensor $\mathcal{P}^{\mu\nu}$ depends only on the vector p^μ so that it is of the form $ag^{\mu\nu} + bp^\mu p^\nu$: the conditions $p^2 = M_W^2$, $p_\mu \mathcal{P}^{\mu\nu} = p_\nu \mathcal{P}^{\mu\nu} = 0$ and $\mathcal{P}^\mu_\mu = -3$ then determine a and b as given in eq. (2.25).

If the boson W has its momentum along the z-axis, $p = (E, 0, 0, p)$ the polarisation vectors are boosted to:

$$\varepsilon^{(1)\mu} = (0, 1, 0, 0), \quad \varepsilon^{(2)\mu} = (0, 0, 1, 0) \quad \text{transverse polarisations} \tag{2.26}$$

$$\varepsilon^{(3)\mu} = \frac{1}{M_W}(p, 0, 0, E) \quad \text{longitudinal polarisation.} \tag{2.27}$$

For a boson W with a momentum making an angle θ in the zOx plane one simply has to make a rotation around the Oy axis, $p = (E, p\sin\theta, 0, p\cos\theta)$, and the polarisation vectors become:

$$\varepsilon^{(1)\mu}(p) = (0, \cos\theta, 0, -\sin\theta), \quad \varepsilon^{(2)\mu}(p) = (0, 0, 1, 0) \quad \text{transverse polarisations} \tag{2.28}$$

$$\varepsilon^{(3)\mu}(p) = \frac{1}{M_W}(p, p\sin\theta, 0, E\cos\theta) \quad \text{longitudinal polarisation.} \tag{2.29}$$

In the high energy limit, in the frame of eqs. (2.22), $E \simeq p \simeq \sqrt{s}/2 \gg M_W$, the longitudinal polarisation vector simplifies to:

$$\varepsilon^{(3)\mu}(p) \approx \frac{1}{M_W}(\sqrt{s}/2, \sqrt{s}/2\sin\theta, 0, \sqrt{s}/2\cos\theta) \approx \frac{p^\mu}{M_W}. \tag{2.30}$$

We use this approximation in the calculaion below. For convenience we introduce the notation

$$\varepsilon^{(1)\mu} \text{ or } \varepsilon^{(2)\mu} = \varepsilon^\mu_T, \text{ and } \varepsilon^{(3)\mu} = \varepsilon^\mu_L.$$

In contrast, we recall that a massless spin 1 particle has only two states of transverse polarisation.

• **Production of massive vector bosons**

After all these kinematic preliminaries we turn to the evaluation of the matrix element. Remembering the $\gamma_\mu(1-\gamma_5)$ coupling of the W boson to the fermions, the matrix element for the diagram above with the electron exchange is:

$$\mathcal{M}^{ij} = -2ig_W^2 \frac{\bar{v}(p_2)\ \not\varepsilon^{(j)}(p_4)(\not p_1 - \not p_3)\ \not\varepsilon^{(i)}(p_3)(1-\gamma_5)u(p_1)}{(p_1-p_3)^2} \tag{2.31}$$

where we have pushed the $(1-\gamma_5)$ factors to the right, hence the factor 2. Without doing the calculation explicitely one can guess that the matrix element squared will contain terms of the form :

$$|\mathcal{M}^{ij}|^2 \quad \propto \quad \frac{g_W^4}{((p_1-p_3)^2)^2} \{(p_1.\varepsilon^{(i)}(p_3)\ p_2.\varepsilon^{(j)}(p_4))^2, \cdots,\ (p_k.p_l)\ (p_1.\varepsilon^{(j)}(p_3))^2\ \varepsilon^{(j)^2}(p_4), \cdots,$$
$$(p_k.p_l)\ (p_m.p_n)\ (\varepsilon^{(i)}(p_3).\varepsilon^{(j)}(p_4))^2,\ \cdots\}, \tag{2.32}$$

with $p_k, p_l, \cdots$ any of the external momenta. In the limit $\sqrt{s} \gg M_W$, it is easy to see that, if both polarisation vectors are transverse, all expressions such as:

$$p_1.\varepsilon_T(p_3)\ p_2.\varepsilon_T(p_4) \propto (p_1.\varepsilon_T(p_3))^2 \propto s, \tag{2.33}$$

since all components of the transverse polarisation vectors are of order 1 and the momenta are generically of order $\sqrt{s}$. If, on the contrary, both $W's$ are longitudinally polarised, the components of the polarisation vectors being of $\mathcal{O}(\sqrt{s}/M_W)$ one finds:

$$p_1.\varepsilon_L(p_3)\ p_2.\varepsilon_L(p_4) \propto ((p_1.\varepsilon_L(p_3))^2 \propto \frac{s^2}{M_W^2}. \tag{2.34}$$

In consequence ($p_1.p_3 \propto s$),

$$|\mathcal{M}_{TT}|^2 \propto g_W^4, \qquad \text{and} \qquad |\mathcal{M}_{LL}|^2 \propto g_W^4 \frac{s^2}{M_W^4}. \tag{2.35}$$

Asymptotically the matrix element squared for the production of longitudinal bosons grows very fast while it is bounded in the case of transverse bosons. Since integrating over phase space to obtain the total cross section brings a factor $1/s$ (see eq. (2.11)) we expect the production of two longitudinal W's to violate unitarity. To calculate effectively this cross section, one has to be a bit more refined and to go back to eq. (2.31) using the form eq. (2.30) for the polarisation vectors:

$$\mathcal{M}_{LL} = -i\frac{g_W^2}{M_W^2}\bar{v}(p_2)\ \not p_4 \left[\frac{\not p_4 - \not p_2}{(p_2-p_4)^2} + \frac{\not p_1 - \not p_3}{(p_1-p_3)^2}\right] \not p_3(1-\gamma_5)u(p_1), \tag{2.36}$$

where we have used the trivial equality $p_1 - p_3 = p_4 - p_2$. Anticommuting the matrices so as to bring $\not p_1$ close to $u(p_1)$ and use the Dirac equation $\not p_1 u(p_1) = 0$ and similarly for $\not p_2$ and $\bar{v}(p_2)$ we end up with:

$$\mathcal{M}_{LL} = -i\frac{g_W^2}{M_W^2}\bar{v}(p_2)\left[\frac{M_W^2-2p_2.p_4}{(p_2-p_4)^2}\not{p}_3+\not{p}_4\frac{2p_1.p_3-M_W^2}{(p_1-p_3)^2}\right](1-\gamma_5)u(p_1)$$

$$= -i\frac{g_W^2}{M_W^2}\bar{v}(p_2)\left[\not{p}_3-\not{p}_4\right](1-\gamma_5)u(p_1) \qquad (2.37)$$

Averaging on the initial polarisations one finds:

$$\overline{\Sigma}|\mathcal{M}_{LL}|^2 = \frac{1}{4}\frac{g_W^4}{M_W^4}2\,\mathrm{Tr}(\not{p}_2(\not{p}_3-\not{p}_4)\,\not{p}_1(\not{p}_3-\not{p}_4)(1-\gamma_5))$$

$$= 2\frac{g_W^4}{M_W^4}\mathrm{Tr}(\not{p}_2\,\not{p}_3\,\not{p}_1\,\not{p}_3(1-\gamma_5))$$

$$= 16\frac{g_W^4}{M_W^4}\,p_1.p_3\,p_2.p_3 = \frac{g_W^4}{M_W^4}\,s^2(1-\cos^2\theta), \qquad (2.38)$$

in the limit $s \gg M_W^2$. It is then easy to obtain the differential cross section using the phase space factor of eq. (2.11) and then the integrated cross section for $\nu_e\,\bar{\nu}_e \to W_L\,W_L$:

$$\sigma(\nu_e\bar{\nu}_e \to W_L^+W_L^-) = \frac{g_W^4}{24\pi}\frac{s}{M_W^4}. \qquad (2.39)$$

In contrast one can estimate the cross section of $\nu_e\,\bar{\nu}_e \to W_T\,W_T$ (but it is more tedious and is left as an exercise):

$$\sigma(\nu_e\bar{\nu}_e \to W_T^+W_T^-) \propto \frac{g_W^4}{M_W^2} \qquad \text{when } s \to \infty. \qquad (2.40)$$

We thus find that the production of longitudinally polarised vector bosons violates the unitarity limit while that of transverse bosons is well behaved at high energies. Several ways have been tried to cure this problem: among them one can mention the hypothesis of a new heavy lepton (fig. 1b) and choose its couplings to enforce a proper behaviour of the cross section at high energies. It turns out that

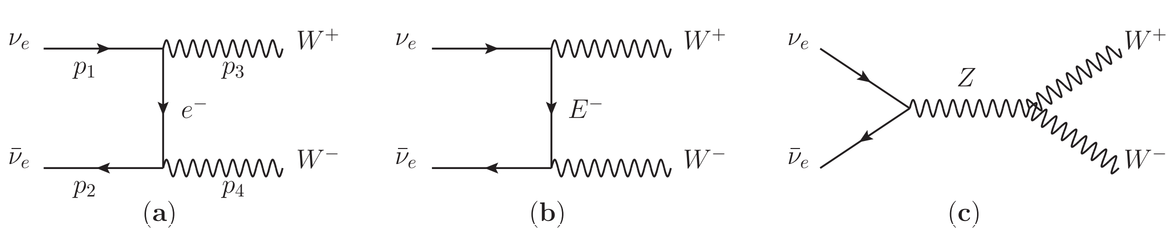


Figure 1: *Possible Feynman diagrams for $\nu_e\overline{\nu}_e \to W^+W^-$ scattering.* **(a)**: *e exchange;* **(b)** *hypothetical heavy electron E exchange;* **(c)** *neutral vector boson Z exchange.*

another possibility, namely that of a heavy neutral vector boson, denoted Z (fig. 1c), is realised in

Nature. Assuming the Z coupling to the fermions of type $g_Z \gamma_\mu (a - b\gamma_5)$ and to the charged W bosons of type[5] $g'_Z (p^\rho_{W^+} - p^\rho_{W^-}) g^{\mu\nu} + \cdots$, they can be chosen to make cross sections such as $\nu_e\, \bar{\nu}_e \to W^+\, W^-$, $e^+\, e^- \to W^+\, W^-$, $\cdots$ asymptotically well behaved. However, this patch up job is not yet sufficient to have a satisfactory model. Indeed, keeping fermion masses and considering for exemple $e^+\, e^- \to Z\, Z$ scattering one finds an interference piece in the cross section $\sim\ g^4 m_e \sqrt{s}/M_Z^4$ which again violates unitarity! Similar problems arise in the $W\, W$ scattering process, *e.g.* $W^+\, W^- \to W^+\, W^-$ which are studied at LHC or will be in the future e^+e^- high energy linear colliders: the cross sections for these processes diverge linearly in s. These problems can be solved by supposing the existence of a scalar particle which interacts with the bosons as well as the fermions with appropriately chosen couplings.

One can thus construct a viable electroweak theory in the pedestrian way described above, carefully choosing masses and couplings of the newly introduced particles so as to ensure the correct behaviour of all cross sections. It is more instructive however to assume that these relations among masses and couplings arise from some symmetry property. This is what is done next. Before doing that, one should discuss the implications of the $\gamma_\mu (1 - \gamma_5)$ coupling in the weak interactions compared to the γ_μ coupling of electrodynamics Then we describe in some details the symmetry group assuming global then local gauge invariance. At this level, the chosen group requires all fields to be massless. The theory is renormalisable (well behaved at asymptotic energies) being a non-abelian field theory. Then, by the mechanism of "spontaneous symmetry breaking" whereby the symmetry of the lagrangian is preserved but the choice of a ground state breaks the symmetry, fermions and gauge bosons acquire a mass. After symmetry breaking, the theory remains renormalisable as a consequence of the underlying gauge invariance which imposes the required relations between couplings. One is left however with a large number of parameters (at least 18 for the Standard Model with massless neutrinos and 25 with massive neutrinos) which gives a motivation for a (still unsuccessful!) search of a deeper symmetry.

[5]Dimensional arguments and gauge invariance lead to such a choice.